Parametric Option Pricing with Chebyshev Interpolation

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Stochastic Methods in Physics and Finance:
July 24, 2015, Crete
Day-to-day tasks in the financial industry: calibration, risk assessment, quotation

Respecting credit and liquidity risk results in nonlinear pricing equations, BSDEs and PIDEs. However, today...

..."it’s impossible to implement on large portfolios in an efficient way"...

D. Brigo, Conference Munich 2015

We need reliable complexity reduction techniques.
We exploit the recurrent nature of the option pricing problem in an efficient, reliable and general way.

Fast Fourier Transform
for plain vanilla prices — for varying strikes $K$.

Carr, Madan (1999), Raible (2000),
Lee (2004), Lord, Fang, Bervoets and Oosterlee (2008),
Feng and Linetsky (2008), Kudryavtsev and Levendorskiy (2009),
Boyarchenko and Levendorskiy (2014), . . .
Parametric Option Pricing $= \text{POP}$

N = 1

Figure: Call prices in the BS model
Parametric Option Pricing = POP

Figure: Call prices in the BS model
Parametric Option Pricing = POP

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Parametric Option Pricing $= \text{POP}$

$N = 6$

Figure: Call prices in the BS model
Parametric Option Pricing = POP

Figure: Call prices in the BS model
Parametric Option Pricing = POP

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Parametric Option Pricing = POP

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Figure: Call prices in the BS model
Parametric Option Pricing = POP

Crucial observations:

- The same pricing problem needs to be solved for a large set of different parameter values: POP Parametric Option Pricing.
- Model prices typically are continuous or even smooth in the parameter values.

We exploit the functional dependence

\[ x \mapsto Price^x, \quad x \equiv \text{parameters} \]

and approximate this function by interpolation.

Related literature: Interpolation of Black Scholes’s call prices in the volatility, Pistorius, Stolte (2012)
Parametric Option Pricing = POP

Figure: Call prices in the CGMY model
Chebyshev Interpolation
Chebyshev Interpolation Points

Chebyshev nodes for $D=1$
Chebyshev Polynomial Interpolation for POP

We approximate

\[ K \mapsto Price^K = E[f^K(X)] \]

by **Chebyshev polynomial interpolation** for \( K = [-1, 1] \)

\[ Price^K \approx 2 \sum'_{0 \leq j \leq N} c_j(Price) T_j(K), \]

with

- **nodes** \( K_k := \cos\left(\pi \frac{2k+1}{2N+2}\right) \)
- **Chebyshev polynomials** \( T_j(K) := \cos\left(j \arccos(K)\right) \)
- **coefficients** \( c_j(Price) := \sum'_{0 \leq k \leq N} Price^{K_k} \cos\left(j\pi \frac{2k+1}{2N+2}\right). \)

\( \sum' \) indicates that the first summand is multiplied with 1/2.
Accuracy of Chebyshev Interpolation

By implementing Chebyshev interpolation Platte & Trefethen (2008) aim . . . “to combine the feel of symbolics with the speed of numerics.”

\[
\mathbb{R}\ni \rho = a + b
\]

Theorem (Bernstein (1912), Trefethen (2013))

If \( g : [-1, 1] \rightarrow \mathbb{R} \) is analytic and bounded in the ellipse with foci \( \pm 1 \) and semiaxis lengths summing to \( \varrho \), then for \( I_N(g)(x) := \sum_{0 \leq j \leq N} c_j(g) T_j(x) \),

\[
\|g - I_N(g)\|_\infty \leq C \frac{\varrho^{-N}}{\varrho - 1}.
\]
Tensorized Chebyshev Interpolation

Chebyshev nodes for $D=2$

Figure: A set of $d$-variate Chebyshev points $\hat{x}_i$ for $d=2$ and $N=20$ as defined in (?). Comparing the arrangement of points $\hat{x}_i$ for $d=2$ to the arrangement for $d=1$ as shown in Figure ??, the tensorizing structure is clear to see.

The basis functions $T_j$, $j \in \mathbb{N}^d_0$, possess a tensorizing structure, and can thus be expressed by their univariate siblings defined in (?),

$$T_j(x_1, \ldots, x_d) = \prod_{i=0}^{d-1} T_{j_i}(x_i).$$
Tensorized Chebyshev Interpolation

We approximate

\[ x = (K, T, p) \mapsto Price^{K, T, p} := E[f^K(X^p_T)] \]

by tensorized Chebyshev polynomial interpolation for \( x \in [-1, 1]^D \)

\[ I_{N_1 \ldots N_D}(Price)(x) := \sum_{j_1=0}^{N_1} \ldots \sum_{j_D=0}^{N_D} c_{(j_1, \ldots, j_D)} \prod_{i=1}^D T_{j_i}(x_i), \]

with coefficients

\[ c_{(j_1, \ldots, j_D)} = \left( \prod_{i=1}^{D} \frac{2^1 \{j_i > 0\}}{(N_i + 1)} \right) \sum_{k_1=0}^{N_1} \ldots \sum_{k_D=0}^{N_D} Price_x^{(k_1, \ldots, k_D)} \prod_{i=1}^D \cos \left( j_i \pi \frac{2k_i + 1}{2N + 2} \right). \]
Tensorized Chebyshev Interpolation

For $\varrho = (\varrho^1, \ldots, \varrho^D) \in (1, \infty)^D$ let the tensorized Bernstein ellipse

$$B([-1, 1]^D, \varrho) = B([-1, 1]^D, \varrho^1) \times \ldots \times B([-1, 1]^D, \varrho^D).$$

**Theorem (Gaß, G., Mahlstedt, Mair (2015))**

If $g$ has an analytic and bounded extension to $B([-1, 1]^D, \varrho)$ for some parameter vector $\varrho \in (1, \infty)^D$, then

$$\|g - I_N(g)\|_\infty \leq C \varrho^{-N},$$

where $\varrho = \min_{1 \leq i \leq D} \varrho_i$ and $N = \min_{1 \leq i \leq D} N_i$.

Proof by a generalization of the univariate case by induction over $D$. 
Tensorized Chebyshev Interpolation

The error decay is of order $O\left(\varrho^{M^{1/D}}\right)$ in the number $M$ of degrees of freedom.

The result naturally extends to functions

$$f : \mathcal{P} \rightarrow \mathbb{R}$$

with hyperrectangular domain $\mathcal{P}$. We call $B(\mathcal{P}, \varrho)$ the appropriately transformed tensorized Bernstein ellipse.
Application to POP
Chebyshev Interpolation Applied to POP

Precomputation phase:
Determine the approximate option prices

\[ \text{Price}^x = (K, T, p) \]

at the Chebyshev nodes \( x^{k_1, \ldots, k_D} = (x_{k_1}^1, \ldots, x_{k_D}^D) \) e.g. with

- Fourier techniques
- Partial (integral) differential equation methods
- Monte Carlo

---

Evaluation phase:
Evaluate the approximate polynomial

\[
I_{N_1 \ldots N_D}(\text{Price})(x) := \sum_{j_1=0}^{N_1} \ldots \sum_{j_D=0}^{N_D} c_{(j_1, \ldots, j_D)} \prod_{i=1}^{D} T_{j_i}(x_i),
\]

with explicit coefficients.
Chebyshev Interpolation Applied to POP

**Figure**: Call prices in a CGMY model, $N_1 = N_2 = 15$ Chebyshev nodes
Chebyshev Interpolation Applied to POP

Figure: Error plot for $N_1 = N_2 = 15$, call prices in a CGMY model
Chebyshev Interpolation Applied to POP

Figure: Price and error plot, N=20, call prices in a CGMY model.
Conditions on Options and Models for (Sub)Exponential Convergence
Convergence of Chebyshev Interpolation for POP

Task: Determine **accessible sufficient conditions** for

\[ p = (K, T, \pi) \mapsto Price^{K,T,\pi} := E[f^K(X^\pi_T)] \]

for \( p = (K, T, \pi) \in \mathcal{P} \) to have an **analytic bounded extension** to some generalized Bernstein ellipse.

Observe in \( Price^{K,T,p} = E[f^K(X^p_T)] \) typically

\[ K \mapsto f^K(X^p_T) \quad \text{e.g.} \quad K \mapsto (X^p_T - K)^+ \]

is **not even differentiable**.
Convergence of Chebyshev Interpolation for POP

Our approach: **Fourier representation** of option prices,

\[ \text{Price}^{K,T,p} = \frac{1}{(2\pi)^d} \int_{\mathbb{R}^d + i\eta} \hat{f}^K(-z) \varphi^{T,p}(z) \, dz \]

with the characteristic functions \( \hat{f}^K \) of \( f^K \) and \( \varphi^{T,p} \) of \( X_T^p \).
Conditions

Let the parameter set $\mathcal{P} = \mathcal{P}^1 \times \mathcal{P}^2 \subset \mathbb{R}^D$ be a \textbf{hyperrectangle}. Let $\varrho \in (1, \infty)^D$ with $\varrho^1 := (\varrho_1, \ldots, \varrho_m)$ and $\varrho^2 := (\varrho_{m+1}, \ldots, \varrho_D)$ and let weight $\eta \in \mathbb{R}^d$.

\begin{enumerate}[(A1)]
    \item \textbf{Integrability} of $x \mapsto e^{\langle \eta, x \rangle} f^K(x)$ for all $K \in \mathcal{P}^1$.
    \item \textbf{Analyticity} of $K \mapsto \hat{f}^K(z - i\eta)$ in $B(\mathcal{P}^1, \varrho^1)$ for all $z \in \mathbb{R}^d$.
    \item \textbf{Exponential moment} $E\left(e^{-\langle \eta, X_T^p \rangle}\right) < \infty$ for all $(T, p) \in \mathcal{P}^2$.
    \item \textbf{Analyticity} of $(T, p) \mapsto \varphi^{T,p}(z + i\eta)$ in $B(\mathcal{P}^2, \varrho^2)$ for all $z \in \mathbb{R}^d$.
    \item \textbf{Uniform bound}: There exists $h \in L^1(\mathbb{R}^d)$ such that \[ \max_{(K,T,p) \in B(\mathcal{P},\varrho)} \left| \hat{f}^K(-z - i\eta)\varphi^{T,p}(z + i\eta) \right| \leq h(z) \quad \text{for all } z \in \mathbb{R}^d. \]
\end{enumerate}
Convergence of Chebyshev Interpolation for POP

\[ Price^{K,T,p} = E[f^K(X^p_T)] \]

**Proposition (Eberlein, G., Papapantoleon (2010))**

Assume (A1), (A3) and (A5). Then each \( Price^{K,T,p} = E[f^K(X^p_T)] \) has the Fourier representation

\[
Price^{(K,T,p)} = \frac{1}{(2\pi)^d} \int_{\mathbb{R}^d + i\eta} \hat{f^K}(-z) \varphi^{T,p}(z) \, dz.
\]
Theorem (Gaß, G., Mahlstedt, Mair (2015))

Let \( \varrho \in (1, \infty)^D \) and weight \( \eta \in \mathbb{R}^d \). Conditions (A1)–(A5) imply

\[
\max_{(K, T, p) \in \mathcal{P}} |\text{Price}^{(K, T, p)} - I_N(\text{Price}^{(\cdot)})(K, T, p)| \leq C \varrho^{-N},
\]

where \( \varrho = \min_{1 \leq i \leq D} \varrho_i \) and \( N = \min_{1 \leq i \leq D} N_i \).
Examples investigated in more detail

Models

- Black Scholes, Merton, NIG, CGMY,…
- time-inhomogeneous Lévy
- Heston, affine (jump) models

Payoff profiles

- European: call, put, digital down&out, power,…
- Baskets: call on basket, call on minimum,…
Detailed analysis in Lévy and affine models
European options in Lévy models

\( S_t^p = S_0 e^{L_t^p} \) with \( L^p \) a Lévy process and special semimartingale for varying parameters \( p = (b, \sigma) \)

\[
E(e^{izL_t^p}) = e^{t\psi_p(z)},
\]

\[
\psi_p(z) = \frac{\sigma^2 z}{2} + ibz + \int_{\mathbb{R}} \left( e^{izx} - 1 - izx \right) F(dx).
\]

Martingale condition

\[
b = b(r, \sigma) = r - \frac{\sigma^2}{2} - \int_{\mathbb{R}} \left( e^x - 1 - x \right) F(dx).
\]

Fair value of a European option

\[
\text{Price}(r,K,S_0,T,p) = e^{-rT} E(f^K(S_0 e^{L_T^p})).
\]
European options in Lévy models

Let \( P = [K, \overline{K}] \times [T, \overline{T}] \times [b, \overline{b}] \times [\sigma, \overline{\sigma}] \) with \( \overline{T} > 0 \).

Corollary (Gaß, G., Mahlstedt, Mair (2015))

Let \( \eta \in \mathbb{R}, \rho^1 > 1 \). Assume (A2), that \( \int_{|x| > 1} (e^{-\eta x} \vee e^x) F(dx) < \infty \) and

\[
\text{(A1') } \sup_{K \in B([K, \overline{K}], \varrho^1)} \left\| e^{\langle \cdot, \eta \rangle} f^K \right\|_{L^1(\mathbb{R}^d)} < \infty \text{ and } \left| \hat{f}^{p^1} (-z - i\eta) \right| \leq c_1 e^{c_2 |z|}
\]

for some \( c_1, c_2 > 0 \).

If additionally

(i) \( \sigma > 0 \), or

(ii) there exist \( \alpha \in (1, 2] \) and \( C_1, C_2 > 0 \) such that

\[
\Re(\tilde{\psi})(z + i\eta) \leq C_1 - C_2 |z|^{\alpha} \quad \text{for all } z \in \mathbb{R},
\]

then there exist constants \( C > 0 \) and \( \varrho > 1 \) such that for \( N = \min_{1 \leq i \leq 4} N_i \),

\[
\max_{(K, T, b, \sigma) \in P} \left| \text{Price}^{(K, T, b, \sigma)} - l_N(\text{Price}^{(\cdot)})(K, T, b, \sigma) \right| \leq C \varrho^{-N}.
\]
Basket Options in Affine Models

Parametrized affine process $X^{\pi'}$ with state space $\mathcal{D} \subset \mathbb{R}^d$, $\pi' \in \Pi'$ and with $\mathbb{C}$-valued $\varphi^{\pi'}$ and $\mathbb{C}^d$-valued $\phi^{\pi'}$ solutions of generalized Ricatti equations satisfying

$$
\varphi(t,x,\pi')(z) = E\left( e^{i\langle z, X_{t}^{\pi'} \rangle} \mid X_{0}^{\pi'} = x \right) = e^{\varphi^{\pi'}(t,iz)+\langle \phi^{\pi'}(t,iz),x \rangle},
$$

for every $t \geq 0$, $z \in \mathbb{R}^d$ and $x \in \mathcal{D}$.

Fair value of a European option

$$
Price^{(K,T,x,\pi')} = E\left( f^K(X_T^{\pi'}) \mid X_0^{\pi'} = x \right)
$$

where $f^K$ is a parametrized family of measurable payoff functions $f^K : \mathbb{R}^d \to \mathbb{R}_+$ for $K \in \mathcal{P}^1$. 

Corollary (Gaß, G., Mahlstedt, Mair (2015))

Assume (A1'), (A2), (A3') for \( \eta \in \mathbb{R}^d \) and hyperrectangular \( P \subset \mathbb{R}^D \). Let

(i) for every \( p^2 = (t, x, \pi') \in P^2 \subset \mathbb{R}^{D-m} \) and every \( z \in \mathbb{R} + i\eta \),

\[
\varphi_{p^2=(t,x,\pi')}(z) = E\left( e^{i\langle z, X_t^{\pi'} \rangle} \mid X_0^{\pi'} = x \right) = e^{\phi^{\pi'}(t,iz)+\langle \psi^{\pi'}(t,iz), x \rangle},
\]

(ii) \( (t, \pi') \mapsto (\phi^{\pi'}(t, iz - \eta), \psi^{\pi'}(t, iz - \eta)) \) has an analytic extension to a Bernstein ellipse \( B(\Pi', \varrho') \) with \( \varrho' \in (1, \infty)^{D-m} \) for every \( z \in \mathbb{R}^d \),

(iii) there exist \( \alpha \in (0, 2] \) and \( C_1, C_2 > 0 \) such that for every \( p^2 = (t, x, \pi') \in P^2 \) and every \( z \in \mathbb{R} \),

\[
\Re\left( \phi^{\pi'}(t, iz) + \langle \psi^{\pi'}(t, iz), x \rangle \right)(z + i\eta) \leq C_1 - C_2|z|^\alpha.
\]

Then there exist constants \( C, \varrho > 0 \) such that

\[
\max_{p \in P^1 \times P^2} |Price^p - l_N(Price^{(\cdot)})(p)| \leq C\varrho^{-\min_{1 \leq i \leq D} N_i}.
\]
Numerical Results
### Numerical Experiments: European Call

Fixed strike, $K = 1$

<table>
<thead>
<tr>
<th></th>
<th>$p^2$</th>
<th>$K \in$</th>
<th>$T \in$</th>
</tr>
</thead>
<tbody>
<tr>
<td>BS</td>
<td>$\sigma = 0.2$</td>
<td>[1, 10]</td>
<td>[0.5, 2]</td>
</tr>
<tr>
<td>Merton</td>
<td>$\sigma = 0.15$, $\alpha = -0.04$, $\beta = 0.02$, $\lambda = 3$</td>
<td>[1, 10]</td>
<td>[0.5, 2]</td>
</tr>
<tr>
<td>CGMY</td>
<td>$C = 0.6$, $G = 10$, $M = 28$, $Y = 1.1$</td>
<td>[1, 10]</td>
<td>[0.5, 2]</td>
</tr>
<tr>
<td>Heston</td>
<td>$T = 2$, $\kappa = 1.5$, $\theta = 0.2^2$, $\sigma = 0.25$, $\rho = 0.1$</td>
<td>[1, 10]</td>
<td>$\nu_0 \in [0.1^2, 0.4^2]$</td>
</tr>
</tbody>
</table>
Numerical Experiments: European Call

Figure: Absolute pricing error. We compare the Chebyshev interpolation with $N_1 = N_2 = 20$ to classic Fourier pricing by numerical integration.
Numerical Experiments: European Call

Figure: Convergence study for the BS, Merton, CGMY, Heston model
**Figure**: Convergence study for prices for the BS, Merton, CGMY, Heston
Numerical Experiments: Basket, Lookback, Barrier

Basket option for $d$ underlyings

\[ f(S_1(T), \ldots, S_d(T)) = \left( \left( \frac{1}{d} \sum_{j=1}^{d} S_j(T) \right) - K \right)^+ . \]

Lookback option for $d$ underlyings, $\bar{S}_j(T) := \max_{t \leq T} S_j(t)$,

\[ f(S(t)_{0 \leq t \leq T}) = \left( \left( \frac{1}{d} \sum_{j=1}^{d} \bar{S}_j(T) \right) - K \right)^+ . \]

Barrier option for $d$ underlyings, down&out, $\underline{S}_j(T) := \min_{t \leq T} S_j(t)$,

\[ f(S(t)_{0 \leq t \leq T}) = \left( \left( \frac{1}{d} \sum_{j=1}^{d} \underline{S}_j(T) \right) - K \right)^+ \cdot 1_{\{\underline{S}_j(T) \geq 80, j=1, \ldots, d\}} . \]
Numerical Experiments: Basket, Lookback, Barrier, American Put

\( r = 0, \text{ Free parameters } K \in [83.33, 125] \text{ and } T \in [0.5, 2] \)

<table>
<thead>
<tr>
<th>Model</th>
<th>fixed parameters:</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( p^1 )</td>
<td>( p^2 )</td>
</tr>
<tr>
<td>BS</td>
<td>( S_{j,0} = 100, )</td>
<td>( \sigma_j = 0.2 )</td>
</tr>
<tr>
<td>Heston</td>
<td>( S_{j,0} = 100, )</td>
<td>( \kappa_j = 2, )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \theta_j = 0.2^2, )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \sigma_j = 0.3, )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \rho_j = -0.5, )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \nu_{j,0} = 0.2^2 )</td>
</tr>
<tr>
<td>Merton</td>
<td>( S_{j,0} = 100, )</td>
<td>( \sigma_j = 0.2, )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \alpha_j = -0.1, )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \beta_j = 0.45, )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \lambda_j = 0.1 )</td>
</tr>
</tbody>
</table>

Monte Carlo simulation of 1 Mio sample paths, antithetic variates, 400 time steps per year. 95\% confidence bound for a price level of 10.
Numerical Experiments: Basket, Lookback, Barrier, American Put

\( N = 30 \) Chebyshev nodes in each varying parameter

<table>
<thead>
<tr>
<th>Model</th>
<th>Option</th>
<th>( d )</th>
<th>( \varepsilon_{L_{\infty}} )</th>
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<th>CB price</th>
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<td>Basket</td>
<td>5</td>
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<td>10.2988</td>
<td>10.2986</td>
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<tr>
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<td>Merton</td>
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<td>( 2.4 \cdot 10^{-3} )</td>
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<td>BS</td>
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<td>18.7975</td>
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<tr>
<td>Merton</td>
<td></td>
<td></td>
<td>( 5 \cdot 10^{-3} )</td>
<td>21.275</td>
<td>21.28</td>
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<tr>
<td>BS</td>
<td>Barrier</td>
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<td>10.0149</td>
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<tr>
<td>Heston</td>
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<td></td>
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<td>5.4898</td>
<td>5.4942</td>
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<tr>
<td>Merton</td>
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<td></td>
<td>( 7.0 \cdot 10^{-3} )</td>
<td>9.2406</td>
<td>9.2476</td>
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<tr>
<td>BS</td>
<td>American</td>
<td>1</td>
<td>( 8.6 \cdot 10^{-3} )</td>
<td>9.5515</td>
<td>9.5602</td>
</tr>
</tbody>
</table>
Numerical Experiments: Digital Down&Out

Figure: Absolute pricing error for a European digital down&out option
Applications and further developments
Benefits

**Accurate Acceleration** in models with low number of parameters

- **Pricing**
  Precomputation of prices once with a fine resolution, then always a fast realization available.

- **Calibration**
  Interpolation of the objective function for fast optimization.

- **Hedging, risk assessment, parameter uncertainty,**...

---

**Gain of freedom in modeling** with a low number of parameters:
From ”For liquid options a fast pricer (Fourier-representation) is necessary.”
To ”An accurate pricer is available and the parameter dependence is analytic.”
Thus one may include more realistic features, e.g. dividends.
Contribution

• Propose Chebyshev interpolation for parametric option pricing
• Show that the method is applicable for a variety of models and option types.
  (a) Theoretically:
    - Exponential convergence for analytic parameter dependence.
    - Explicit conditions on the Fourier transform of payoff and distribution.
    - Examples
  (b) Numerically:
    - Experimental order of convergence for Call and digital in BS, Heston, Merton, CGMY model. (CI combined with Fourier method)
    - (Path-dependent) multivariate options: CI combined with Monte-Carlo and Finite Differences.
Outlook
Limitations of Chebyshev Interpolation

Methodological constraints

- Curse of dimensionality: degree of freedom of order $N^D$ for $D$-dimensional parameter space
- Requirement: Tensor structure of parameter space!

Realistic requirements

- Dimension of the parameter space is typically large.
- Parameter constraints often do not result in hyperrectangulars, e.g. entries covariance matrices.

How to find a generic interpolation that is flexible in the parameter space?
Literature


