Adaptive approximation versus tree approximation

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Outline

Approximating gradients

Let

- $\Omega \subset \mathbb{R}^d$ be a domain,
- $\mathcal{T}$ a triangulation of Ω generated by bisection,
- $S(\mathcal{T})$ the space of continuous linear finite element over $\mathcal{T}$.

Given a target function $v \in H_0^1(\Omega)$, let

$$E(v, \mathcal{T}) := \inf\{\|v - V\|_\Omega \mid V \in S(\mathcal{T})\}$$

be the best error over $\mathcal{T}$ with

$$\|w\|_\Omega := \left(\int_\Omega |\nabla w|^2\right)^{1/2}.$$

Local error functionals

Given any element T, let

$$e(v, T) := \{\|v - P\| \mid P \in P_1(T)\}$$

be the best error on T.

Because of the trace theorem, there holds

$$\sum_{T \in \mathcal{T}} e(v, T)^2 \leq E(v, \mathcal{T})^2 \leq C_{\text{dec}}^2 \sum_{T \in \mathcal{T}} e(v, T)^2$$

where C_{dec} is bounded in terms of the shape regularity of $\mathcal{T}$.

Outline

Adaptive approximation

Starting from $N := 0$ and an initial triangulation $\mathcal{T}^0$, iterate

- choose a $T \in \mathcal{T}^N$ such that $e(T)$ is maximal in $\mathcal{T}^N$,
- bisect T to obtain $\mathcal{T}^{N+1}$ and increment N.

Goes back to Birman e Solomyak '67, also called greedy or maximum strategy and, in the terminating variant, thresholding.

For example, if $d = 2$ and $v \in W^{2,p}(\Omega)$ with $p > 1$, then

$$E(v, \mathcal{T}^N) \leq CN^{-1/2}$$

(cf. Binev/Dahmen/DeVore)

Tree approximation – marking strategy

Introduce the marking indicators (cf. Binev/DeVore '04)

$$\xi(T) := \begin{cases} e(T) & \text{if } T \text{ has no parent,} \\ \left[e(T)^{-2} + \xi(T')^{-2}\right]^{-1/2} & \text{if } T' \text{ is the parent of } T. \end{cases}$$

Starting from $N := 0$ and the initial triangulation $\mathcal{T}^0$, iterate

- choose $T \in \mathcal{T}^N$ such that $\xi(T)$ is maximal in $\mathcal{T}^N$,
- bisect T to obtain $\mathcal{T}^{N+1}$ and increment N.

Tree approximation – instance optimality

Since

- $e(T_1)^2 + e(T_2)^2 \leq e(T)^2$ for T_1, T_2 children of T,
- $e(T)$ depends only on T (and not its neighboring elements),

there holds

$$E(v, \mathcal{T}^N) \leq 2C_{\mathrm{dec}} \min\{E(v, \mathcal{T}) \mid \#\mathcal{T} \leq N/2\}.$$

(cf. Binev/Dahmen/DeVore/Lamby)

The subadditivity can be relaxed at the price of bigger constants in the quasi-optimality.

Outline

Implementation

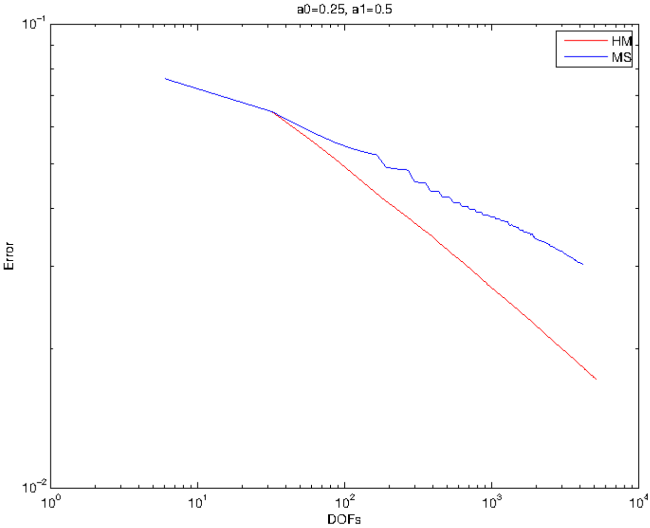
- using ALBERTA by Schmidt/Siebert and collaborators,
- approximating the local error functionals $e(v, T)$ by quadratures rules of order 17, where

$$v(x) = \left|\log(\tfrac{1}{2}|x|)\right|^{\rho}$$

with $0 < \rho < \frac{1}{2}$ such that

$$u \in H^1\big(B(0;1)\big) \setminus W^{2,1}\big(B(0;1)\big).$$

$\rho = 1/4$



$\rho = 3/8$