Anderson Localization of Light and beyond: Disorder-Enhanced Transport

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Outline

• Introduction: intuition on observables
• Anderson Localization: is it or is it not?
• Transverse localization of light
• Experiments
• Some reflections on why do experiments
• Localization under disorder + nonlinearity
• Hyper-transport of light: faster than ballistic?
• Localization in quasi-crystals? De-localization?
• Amorphous photonic media
Drude model for electrical conduction in crystals

- Electrons (shown here in blue) constantly bounce between heavier, stationary, crystal ions (shown in red).
- Current is like a cloud of mosquitoes flying randomly at high velocities, with the cloud drifting in a light wind
- Electrons are like tiny billiard balls ... classical picture
- classical random walk ➔ diffusive transport

Paul Drude, 1900
Quantum (waves) theory of electrons in crystals

F. Bloch, Z. Physik 1928

- Electron waves: amplitude & phase
- Bloch waves (extended states)
- Bands separated by gaps

How would a wave theory yield diffusion? Does scattering from disorder play any role?
Disorder and Anderson Localization

Periodic Potential:

\[ \Psi \]

Bloch waves
(extended states)

\[ V(x) \]

A wave propagates freely through the medium
Ballistic Transport/Diffraction

Disordered Potential:

Localized States
Typical scale \( \xi \)

The wave can remain confined in some region of the potential

Philip W. Anderson, 1958 (Nobel Prize 1977)
Transport in Disordered Lattices

From ballistic transport to diffusion

Anderson’s Prediction

\[ \langle r^2 \rangle \rightarrow \text{Const} \quad (t \rightarrow \infty) \]
Natural assumption: scattering is independent on the scattering direction

Enhanced backscattering!
surface of the scattering medium

Weak localization

Akkermans & Maynard 1985
van Albada & Lagendijk 1985
Wolf & Marret 1985
Kaveh, Rosenbluh & Freud 1986
Etemad, Thompson & Andrejko 1986
Why Localization?

Scattering by impurities generates random walk

Classical Diffusion

But: The wave is **coherently** scattered by defects

- Constructive interference of multiple scatterings
- Higher return probability
- Quantum corrections to diffusion, Localization

<table>
<thead>
<tr>
<th>Dimensionality:</th>
<th>$\xi \sim l$</th>
<th>$\xi \sim l \exp(kl)$</th>
<th>$\frac{1}{\xi}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-D</td>
<td>$l$</td>
<td></td>
<td>Disorder Strength</td>
</tr>
<tr>
<td>2-D</td>
<td>$\exp(kl)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3-D</td>
<td>$\frac{1}{\xi}$</td>
<td>Localization above critical disorder, Phase Transition (Conductor-Insulator)</td>
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Well, why not?

Anderson localization requires a disordered potential which is time invariant and interaction-free

Problems for observing localization in solid state physics:

1. Phonons – the potential varies in time, electrons lose their phase coherence due to inelastic scattering

2. Many-body interactions – Nonlinearity effectively modifies the potential

3. Impossible to view / image / measure the actual wavefunction

However: Localization is a WAVE phenomenon (not quantum)

General for all wave systems! (e.g. Optics)
Localization of Light


- Long coherence length/time – easily generated by laser
- Photons don’t interact directly with each other (and when they do – via nonlinear interactions with atoms - we know how to control the interaction)
- Scattering of light – natural phenomenon (clouds, sugar,…)
- Possibility for direct measurement:
  
  imaging the actual wave-packet into camera!
Localization of Light

Traditional Approach – Transmission/Reflection

Enhanced Back-Scattering

Exponential decay of transmitted intensity

Akkermans, J. Phys. Lett. 1985
Albada & Lagendijk, PRL 1985
Wolf & Maret, PRL 1985
Etemad et al., PRL 1986
Wiersma et al., Nature 1997
Chabanov et al., Nature 2000
Störzer et al., PRL 2006

All experiments in completely random media
no observation of localization in lattices + disorder, as Anderson predicted!
### Localization of Light – Brief Overview

#### EXPERIMENTS IN RANDOM EM STRUCTURES

- Wiersma *et al.*, Nature 1997
- Chabanov *et al.*, Nature 2000
- Störzer *et al.*, PRL 2006

Strong localization effects in transmission through 3-D highly-scattering random medium (no underlying periodicity)

#### EXPERIMENTS IN PHOTONIC LATTICES

- Pertsch, Peschel, Lederer, PRL 2004

Linear & nonlinear dynamics in disordered waveguide arrays

- Schwartz, Bartal, Segev, Fishman, Nature 2007

**First observation of Anderson Localization in any disordered lattices** (done in 2D)

- Lahini, Silberberg, Christodoulides, PRL 2008

Demonstration of localized modes in 1D disordered array and NL effects

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**Experiments with matter waves:** Aspect & Inguscio groups, Nature 2008

3D localization: DeMarco & Aspect groups, 2011
Our motivation (2004):
Experimenting Anderson localization of light in a “photonic crystal” + disorder

Questions:
• Finding a system equivalent to Anderson’s
• Observables: what would we actually see?
• How to introduce static disorder into photonic crystals?
• What can we learn from this?

New ideas & quite a few surprises:
• Localization under nonlinearity (2007)
• Amorphous photonic lattices (2010)
• Anti-localization in quasi-crystals (2011)
• Hyper-transport: (2011) Faster than ballistic! (2012)
Photonic crystals: optical waves in an array of fibers equivalent to electrons in a solid crystal

Array of optical waveguides

Electrons in a solid crystal

refractive index
evanescent overlap

single mode fields

mode amplitude
electronic wavefunction

crystal potential

atoms
Transverse Localization in Photonic Lattices

Consider a beam, propagating along $z$ in a medium with **transverse disorder**, uniform along propagation.

Localization:

**Diffraction**

**Absence of Diffusion in Certain Random Lattices**

Perfect lattice

Disordered lattice
Transverse Localization of Light
Suggested by De Raedt, Lagengijk & de Vries (PRL 1989)

Consider a beam, propagating along \( z \) in a bulk medium with **transverse disorder**, uniform along propagation.

Localization = disorder eliminates diffraction

\[
\vec{E}(x, y, z, t) = A(x, y, z)e^{i(k_z-wt)}\hat{x}
\]

\[
k = \frac{wn_o}{c} \quad A - \text{Slowly varying envelope of CW beam}
\]

\[
\frac{i}{k} \frac{\partial A}{\partial z} = -\frac{1}{2k^2} \nabla_\perp^2 A - \frac{\Delta n(x, y)}{n_0} A
\]

**Optics**

Random refractive index change

\( z \leftrightarrow t \)

\( k \leftrightarrow 1/\hbar \)

\( 1 \leftrightarrow m \)

\( \Delta n/n_0 \leftrightarrow -V(\vec{r}) \)

**Quantum Mechanics**

\[
i\hbar \frac{\partial \Psi(\vec{r}, t)}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \Psi + V(\vec{r})\Psi
\]

**Time-invariant potential**
Objective: Anderson Localization in disordered 2D photonic lattices (Periodic potential with super-imposed disorder)

Transverse wavenumber:

\[ k_\perp \propto \frac{1}{\text{beam width}} \ll k = \frac{\omega n_0}{c} \]

Localization length:

\[ \xi_{2d} = \ell \times \exp \left( \frac{\pi}{2} k_\perp \ell \right) \]

\( \ell \) - mean free path
Determined by fluctuations of \( \Delta n \)

Transverse localization can be observed even with a very weak modulation of refractive index

But: It is a statistical problem (at finite distances):

We must take \textit{ensemble averages} (many realization of disorder)
Transport Dynamics Along Propagation
Studied numerically with typical experimental parameters

Average width $\omega_{\text{eff}}$ [\(\mu m\)]

At a finite distance:
A. Diffusive broadening up to a width $\sim \xi$
B. Absence of diffusion, Localization
Transport Dynamics Along Propagation
Studied numerically with typical experimental parameters

When looking at a finite distance (as in our experiments):

But how can we tell Localization from simply reduced diffusion ???

We have to reveal the transport properties of the lattice! (without looking inside the lattice)

Exponential decay of intensity profile marks the Anderson Localization

\[
\ln\langle I \rangle \sim -x^2
\]

\[
\ln\langle I \rangle \sim -|x|
\]
Objective: demonstrate Anderson Localization in 2D photonic lattices containing disorder
(Periodic potential with super-imposed disorder)

The recipe (what do we need?)

• A structure of periodic refractive index, control over parameters (index modulation, periodicity, …)
• Random perturbations with controlled strength
• Multiple realization of disorder (statistics)
• Invariance along propagation direction (= time-independent potential)

• Patience, quiet, some luck, good computer, understanding wife, comfortable mattress at the lab, ….
Making A Disordered Lattice

Using our optical Induction Technique (Nature 2003):

Interfere 3 beams on a photosensitive anisotropic nonlinear material

Add another beam, passed through a diffuser (=random speckles)

Hexagonal Lattice

Controlled Level of Disorder

• Material translates intensity pattern into an index modulation
• Intensity ratio determines disorder level
• **Probe the lattice with an orthogonally-polarized beam**
• Different position on the diffuser – different realization of disorder (statistics)
Making lattice + disorder: problem

We use a diffuser for generating disorder by a speckled intensity pattern

Narrow speckles diffract

Non-stationary propagation of writing beams
The lattice changes along propagation direction (= time-dependent potential)

Localization effects are destroyed!

Solution:

Construct a ring-shaped angular spectrum, with random phase and amplitude along the ring.
Then:

\[ k_z^2 = k^2 - (k_x^2 + k_y^2) \]

All Fourier components travel with the same velocity (random superposition of Bessel beams)

Z-Independent Disordered Lattice
Experimental Setup

Rotating the diffuser – different realization, same statistical properties

The process is repeated for statistical data
**Experimental Results**

**Measure of Confinement:**
(calculated for intensity distribution at lattice output, 10mm propagation)

\[
P = \frac{\int I^2 dxdy}{\left(\int I dxdy\right)^2} [1/\text{area}]
\]

**Inverse Participation Ratio**

**Averaged Effective Width:**
\[
\omega_{\text{eff}} = \langle P \rangle^{-\frac{1}{2}}
\]

**Averaged IPR:**
\[
\langle P \rangle [10^{-5} \mu m^{-2}]
\]

**Average effective width \( \omega_{\text{eff}} [\mu m] \)**


- **Graphs showing the relationship between relative disorder level and averaged effective width.**
Experimental Observation of Anderson Localization

It’s a statistical problem: must take *ensemble average* (*many realizations of disorder*)

Averaged output intensity cross-section (100 samples) at the lattice output (after 10 mm propagation)

- ‘Perfect’ lattice
- 2.5% Disorder
- 15%
- 45%

\[ \langle I(x) \rangle \]

\[ \text{Gaussian Profile} \quad \text{Diffusion} \quad \text{Exponential Decay} \quad \text{Localization} \]

\[ \text{Numerics} \]

5% 30%

Schwartz et al., Nature 2007
Well, we made the first experimental observation of Anderson localization in a periodic structure containing disorder (Schwartz, Bartal, Fishman, Segev, Nature 2007)

BUT: the concept of Anderson localization is known since 1958. One could study it numerically at great accuracy.

Why would we do such experiments at all?

Because Physics is an experimental science, and experiments always lead to new ideas and often offer surprises!

Let’s look at some new ideas/ unresolved issues / new concepts:

• Localization via disorder + nonlinearity (Nature 2007)
• Bandgap and localization in amorphous photonic lattices (PRL 2010)
• Localization / de-localization in quasi-crystals (Science 2011)
• Hyper-transport of light! (Nature Physics 2012)
Disorder and Nonlinearity
Nonlinearities (Coulomb interactions, nonlinear optical response) may play a fundamental role in disordered systems.

Thus far, the combined effect (NL + disorder) is yet unclear.

By increasing the probe intensity – we modify the local index change of the disordered lattice – we study nonlinear propagation with increasing nonlinearity (focusing) and fixed disorder (15%).

Self-focusing nonlinearity promotes Anderson localization!

\[ \ln(\langle I(x) \rangle) \]
with increasing nonlinearity (focusing) and fixed disorder (15%).

\[ \alpha = \text{NL strength} \]

Schwartz et al., Nature 2007
Localization with Dynamically Evolving Disorder

What would happen if we allow the disorder to evolve?

We do know that localization breaks down,

but what would we find get instead?

Diffusive transport? Sub-diffusion? Something else?

Great Surprise:

Hyper-transport of light: faster than ballistic!

Levi et al., Nature Physics 2012
Localization with stationary disorder

Localization under dynamic disorder

inducing random $z$-variations in a controllable manner

$Z_0 = \text{typical distance of variations, below which the underlying potential (refractive index) is approx constant in } z$

The wider the ring in K-space is, the smaller is $Z_0 \rightarrow$ Faster variations
Hyper Transport of Light (simulation results)

\[ i \frac{\partial \Psi}{\partial t} = -\frac{1}{2m} \nabla^2 \Psi + V(x, y, t) \psi, \]

Free diffraction

\[ i \frac{\partial A}{\partial z} = \frac{1}{2k} \nabla^2 A + \Delta n(x, y, z) A \]

Hyper broadening

Real Space

Free diffraction

Momentum Space

\[ Z_{\text{corr}} = 2\pi (K_r \Delta K / n_0 K_0)^{-1} \]

\[ Z_{\text{corr}} = 1.1 \text{mm} \]

\[ Z_{\text{corr}} = 0.53 \text{mm} \]

\[ Z_{\text{corr}} = 0.4 \text{mm} \]

\[ Z_{\text{corr}} = 10.83 \text{mm} \]

\[ Z_{\text{corr}} = 1.1 \text{mm} \]

\[ Z_{\text{corr}} = 0.53 \text{mm} \]

\[ Z_{\text{corr}} = 0.4 \text{mm} \]

\[ W_{\text{eff}} = 47.74 \mu m \]

\[ W_{\text{eff}} = 136.38 \mu m \]

\[ W_{\text{eff}} = 204.51 \mu m \]

\[ W_{\text{eff}} = 221.85 \mu m \]

\[ K_0 = 12.22 \mu m^{-1} \]

\[ Z_{\text{cor}} = 0.8 \text{mm} \]
Breakdown of Anderson localization due to dynamic disorder

\[ \omega_{\text{eff}}(z) \quad [\mu m] \]

Free diffraction

Diffusion

Localization

Hyper broadening !!!

Simulations results

\[ \pi\omega_{\text{eff}}^2 \quad \text{average area beam area} \]
Evolution of the plane-wave spectrum in disordered systems

\[ \Delta k \,[1/\mu m] \]

\[ \text{Z [mm]} \]

Hyper broadening (effective particles)

Hyper broadening (waves)

Localization (stationary disorder)

Diffraction (ballistic transport)

Experimental Observation plane
Hyper Transport of Light: experimental results

Hyper-Transport

Stationary potential – slow variations - faster variations - fast variations – very fast
Hyper Transport of Light (experimental results)

Hyper-transport: expansion faster than ballistic!
**observations**

1. We demonstrated, theoretically and experimentally, that an evolving random potential can give rise to hyper-transport: transport much faster than ballistic.

2. Hyper-transport is always (inevitably) associated with spectral broadening.

3. Experimental setup for studying waves propagating in a disordered potential which stochastically fluctuates in time – controlling both spatial and “temporal” fluctuations.

4. For disorder with finite bandwidth the wavepacket displays hyper-broadening as long as its momentum falls within the bandwidth of the disorder.

Levi et al., Nature Physics 2012
Question under Investigation:

1. Why does the beam accelerate? Where does the “energy” come from?

2. Condition for hyper transport to occur?

3. Asymptotic behavior of hyper-transport? \((Z \rightarrow \infty)\)

4. Universal Exponent of hyper transport

   \[Beam \; Width \propto z^\alpha, \; \alpha > 1 \text{ what is } \alpha?\]

5. Role of Dimensionality?

Some of these are answered in a model based on classical dynamics
Krivolapov et al., New J. of Phys, 2012
Open Questions

1. Hyper-transport in the Raman-Nath regime:
   Can we break the Correspondence Principle?

2. What is the asymptotic profile of the wavefunction?

3. What happens in weakly / uncorrelated disorder?
   Will hyper broadening continue "forever"?

4. What happens for the non-paraxial case (Helmholtz equation)
   and/or non-scalar case (all polarizations)?

And much more…

We are in a new domain of transport physics!
Comparing Waves and Particles: Simulation Results

Position spread is always accompanied by momentum spread:

\( \lambda = 0.5 \, [\mu m] \)
Simulation Results

Spectrum Width at Saturation

Correspondence – high momentum

localization – difference between particles and waves

Correspondence – high momentum

localization – difference between particles and waves

Spectrum Width

k / max (Ω_z) x 10^4
A classical particle is “stuck” in a strip in momentum space, depending on its initial momentum.

- Particles: transition from the fast to the slow strip is forbidden
- Waves: the transition is phase-matched
Anderson localization in quasi-crystals?

- In QCs, the Bloch modes are not extended states, but “critical states”.
- Will there be Anderson localization in QCs?
- Or maybe “Anti-localization”:
  - adding disorder leading to increased transport, at least in some range of parameters?

Many intriguing fundamental questions …

(L. Levi et al. Science, 2011)
Amorphous photonic lattices

In systems containing disorder:

• Eigen-states are localized (“Anderson states”)
• Transport is suppressed
  (stopped in 1D & 2D, depending on disorder level in 3D)
• Transition from diffusion to Anderson localization

But let’s ask something even more general:

• Take a completely disordered (amorphous) system, so disordered that it lacks Bragg diffraction.
  • Are there any gaps in the spectrum?
  • What about effective mass? Defect States?

See M. Rechtsman et al., PRL 2011
Instead of conclusions: where do we go next?

- Localization in **honeycomb lattices** ("photonic graphene")?
- **Topological insulators & disorder** (Rechtsman, Plotnik Segev)
- Localization with entangled photons? (Silberberg & Christodoulides)
- Localization in complex media:
  - metamaterials, metallic particles (Shalaev & Stockman), …
- **Nonlinear effects** – still pose an enigma (Flach, Fishman):
  - focusing/defocusing at different dispersion regimes
- **Solitons in disordered media** (Segev, Conti, Musslimani)
  - formation and transport properties
- Localization with partially-incoherent light
  - And lots of other ideas ……

Experience reveals:

The most important discovery is ALWAYS unexpected
Summary

Studying universal physics with optical tools makes an impact in optics and beyond.