Discrete Dynamics, Self-Accelerating Solitons and PT Symmetric Lattices in Time


University of Erlangen
Max Planck Institute for the Science of Light, Erlangen, Germany
CREOL, College of Optics and Photonics, University of Central Florida
Friedrich-Alexander-University Erlangen

Erlangen:
• since 1002
• 104 000 inhabitants
• Georg Simon Ohm
• Emmy Noether

university
• since 1743
• ≈ 26 000 students
1. Photon Walks in Discrete Temporal Networks

2. PT symmetry in Photon Networks

3. Losses and Fractal Pattern

4. Self-Accelerating Solitonic Structure

5. Summary
### Classical random walks vs. light walks

<table>
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<tr>
<th>Classical random walk</th>
<th>Light walk</th>
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<tbody>
<tr>
<td><img src="image1.png" alt="Diagram" /></td>
<td><img src="image2.png" alt="Diagram" /></td>
</tr>
<tr>
<td>Evolution in discrete steps $m$</td>
<td>Evolution in discrete steps $m$</td>
</tr>
<tr>
<td>Walker randomly steps to the left or right</td>
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</tr>
<tr>
<td>Incoherent addition (particle-like)</td>
<td>Wave interference</td>
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<tr>
<td>Classical diffusion</td>
<td>Ballistic spreading</td>
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Implementation

Classical random walk

- Galton Board or „bean machine“

Light walk

- Beamsplitter or coupler pyramid („Optical Galton Board“)
  - Too many components
  - Difficult to adjust

Source: Wikimedia Commons

Discreteness in Time

- 2 loops of different length
- connected by a 50/50 coupler

- short loop: step to the left
- long loop: step to the right

Position

Zeit

simple and stable

time discretized

Dynamics of Propagation

start: pulse at roundtrip m=0 and position n=0

short loop
\[ u_{n}^{m+1} = \frac{1}{\sqrt{2}}(u_{n+1}^{m} + i v_{n+1}^{m}) \]
\[ u_{n}^{m} : \text{amplitude in short loop} \]
\[ v_{n}^{m} : \text{amplitude in long loop} \]
long loop
\[ v_{n}^{m+1} = \frac{1}{\sqrt{2}}(v_{n-1}^{m} + i u_{n-1}^{m}) \]
\[ n : \text{position} \]
\[ m : \text{roundtrip} \]

⇒ two-component system = spin-system
Free Propagation

start: single pulse in long loop

co-moving reference frame

⇒ ballistic spreading
(no diffusion)

⇒ asymmetry conserved

perfect agreement

experiment

simulation
Trembling Motion

input: single pulse in the lower loop
⇒ pronounced oscillations between the loops
two component system = particle / antiparticle
⇒ double band structure (like Dirac model)

\[
\begin{pmatrix}
  u_n^m \\
  v_n^m
\end{pmatrix} = \begin{pmatrix}
  u_0 \\
  v_0
\end{pmatrix} \exp\left[-i \Theta m + i Q n\right]
\]

⇒ oscillations between the two bands
⇒ trembling motion Zitterbewegung
Loss and gain

temporal / fiber systems:

⇒ easy access to loss and gain

gain:

• tunable semiconductor optical amplifiers

• up to now used to compensate unavoidable losses

loss:

acusto-optical modulators

tunable and time dependent

$PT$ - symmetric systems

broken connections

fixed

diffusion and fractals
Outline

1. Photon Walks in Discrete Temporal Networks
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$\mathcal{PT}$ – symmetric systems

original idea:

complex potentials

with parity-time ($\mathcal{PT}$) symmetry

$V(x) = V(-x)^*$

parity time

$\left(i \frac{\partial}{\partial t}\right)^* = i \frac{\partial}{\partial (-t)}$

e.g. $V(x) = V_R \cos(kx) + i V_I \sin(kx)$

can (but need not) have real valued eigenvalues

for growing $\text{Im}(V) \Rightarrow$ phase transition towards complex eigenvalues

in optics:

potential $V(x) \Rightarrow$ refractive index $n(x) = n_R(x) + n_I(x)$
Advantage of Discreteness

- Continuous evolution
- Applied all at the same time:
  - Gain / loss
  - Phase potential
  - Coupling

Further complicated by Kramers-Kronig relations

- Discrete evolution
- Completely independent:
  - Gain/loss
  - Phase potential
  - Coupling

Full flexibility, can be transferred to time domain

aktive and passive fibers
connected by a 50/50 coupler
phase modulation: $\pm \varphi_0$

- gain and loss alternate per roundtrip
  ⇒ imaginary potential $V_I$
- periodic phase modulation
  ⇒ real potential $V_R$

\[ \Delta L \]

**gain**

\hspace{1cm} \text{change per roundtrip} \hspace{1cm} \text{loss}

\hspace{1cm} \text{gain or } \frac{1}{\text{loss}} \hspace{1cm} \text{odd roundtrip}

\hspace{1cm} \text{short loop: } u_{n+1}^m = \frac{\sqrt{G}}{\sqrt{2}} (u_{n+1}^m + iv_{n+1}^m) \hspace{1cm} \text{long loop: } v_{n}^m = \frac{1}{\sqrt{G}} (iu_{n-1}^m + v_{n-1}^m) \exp(i\varphi(n))

\hspace{1cm} \text{even roundtrip}

\hspace{1cm} \text{short loop: } u_{n+1}^m = \frac{\sqrt{G}}{\sqrt{2}} (u_{n+1}^m + iv_{n+1}^m) \hspace{1cm} \text{long loop: } v_{n}^m = \frac{1}{\sqrt{G}} (iu_{n-1}^m + v_{n-1}^m) \exp(i\varphi(n))

\hspace{1cm} A. Regensburger et al, Nature 488, 167 (2012)
Phase transition in $\mathcal{PT}$–symmetric temporal systems

band structure

passive
$G=1, \, \phi_0 = 0$

beyond threshold
$G=1.4, \, \phi_0 = 0$
$G=1.4, \, 2\phi_0 = 0.39\pi$

below threshold
$G=1.4, \, 2\phi_0 = 0.41\pi$

fields

energy

Light energy

$\text{total}$
$\text{long}$
$\text{short}$

Step m

Position n

Step m

Position n

Step m

Position n

Step m

Position n
Noninvertible propagation at the exceptional point

Z. Lin et al., PRL 106, 213901 (2011)
Noninvertible propagation - vague explanation

continuous system at the exceptional point:

\[ n(x) = n_0 \left[ \cos(2kx) + i \sin(2kx) \right] = n_0 \exp(2ikx) \]

left propagating wave: \( u_-(x, z) \sim \exp(-ikx) \)

\[ \Rightarrow \text{induced polarisation} \sim u_- * n \sim \exp(+ikx) \sim u_+ \Rightarrow \text{scattering} \]

right propagating wave: \( u_+(x, z) \sim \exp(+ikx) \)

\[ \Rightarrow \text{induced polarisation} \sim u_+ * n \sim \exp(+3ikx) \Rightarrow \text{no scattering} \]

discrete system: same band structure \( \Rightarrow \) same scattering
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Loss and gain

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⇒ easy access to loss and gain

gain:

• tunable semiconductor optical amplifiers
• up to now used to compensate unavoidable losses

loss:

*acusto-optical modulators*

tunable and time dependent

\[ \mathcal{PT} \text{- symmetric systems} \]

broken connections

fixed

diffusion and fractals
Inducing Photon Losses

conservative setup
Inducing Photon Losses

conservative setup
Inducing Photon Losses

conservative setup

lossy setup
Inducing Photon Losses

conservative setup

lossy setup

Will the losses change anything?
Lossy Beamsplitter Pyramid

Beamsplitter implementation of the lossy setup

at every second row:

All beams going to the right are blocked by a measurement
Evolution in the Presence of Photon Losses

Please note the following:

- **Recursion relation for the signal in the loop**
  \[ a_{n}^{m+1} = \frac{i}{2} [a_{n+1}^{m} + a_{n-1}^{m}] \]

- **Round trip**

- **Time step**

\[ a_{n}^{m} = a_{0} \left( \frac{i}{2} \right)^{m} \binom{m}{n} \]

\[ \Rightarrow \text{Pascal's triangle} \]

- **Losses due to the open coupler**

\[ \Rightarrow \text{diffusion in amplitude} \]

\[ \Rightarrow \text{loss makes the system classical (like Galton board)} \]

Lossy System - Linearly Growing Phase -

phase modulator inserted into the loop $ PM $  

$ \Rightarrow $ time dependent phase shift $ \exp(i\alpha n) $  

$ \alpha = \frac{p}{q} \pi $ with $ p, q $ coprime  

• no Bloch oscillations  

• point wise recovery after $ q $ steps  

• diffusive spreading + triangular pattern  

$ \alpha = \frac{2}{5} \pi $  

A. Regensburger et al., PRL107, 233902 (2011).
Lossy System - Linearly Growing Phase -

phase modulator inserted into the loop

\[ \Rightarrow \text{time dependent phase shift } \exp(i\alpha n) \]

\[ \alpha = \frac{p}{q} \pi \quad \text{with } p, q \text{ coprime} \]

• no Bloch oscillations

• point wise recovery after \( q \) steps

• diffusive spreading + triangular pattern

• the bigger \( p \) and \( q \) the more fractal the pattern

\[ \alpha = \frac{9}{26} \pi \]
The Triangular Pattern
A Vague Explanation

\[ \alpha = \frac{p}{q} \pi \text{ with } p, q \text{ coprime} \]

**along the outermost line of the triangle** \( n = \pm m \)

- only a single path possible
- no interference
- signal always present

**points inside the triangle** \( |n| < m \)

- photons can reach every position on \( \left( \frac{m - n}{2} \right) \) multiple of \( m \) paths
- acquire phases with multiples of \( 2\alpha \) on their way
- for \( m = q \): multiple of \( q \) paths interfere
  \( \Rightarrow \) complex unit circle homogenously filled
  \( \Rightarrow \) destructive interference
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Nonlinear Effects in the Conservative Lattice

- back to the conservative system
- erbium doped amplifiers + 4km of fiber
- increasing the power
- nonlinearity: Kerr nonlinearity of the fiber

\[ u_{n+1}^m = \frac{1}{\sqrt{2}^m} [u_{n+1}^m \exp(i \chi |u_{n+1}^m|^2) + i v_{n+1}^m \exp(i \chi |v_{n+1}^m|^2)] \exp[i \varphi(n)] \]

\[ v_{n+1}^m = \frac{1}{\sqrt{2}^m} [i u_{n-1}^m \exp(i \chi |u_{n-1}^m|^2) + v_{n-1}^m \exp(i \chi |v_{n-1}^m|^2)] \]
Discrete Solitons in the Conservative Lattice

Injection of a single pulse into the long loop ⇒ discrete soliton

Discrete soliton:
- Propagates at medium velocity
- Alternates between the loops

Increasing power
Broad Beams with Tunable Mass ⇒ Linear Action

dynamics determined by location in the band structure

upper band: anomalous dispersion = positive mass
⇒ attracted by phase dips

lower band: normal dispersion = negative mass
⇒ repelled by phase dips

$\varphi(n)$

$m_{\text{eff}} > 0$
$m_{\text{eff}} < 0$

Experiment

$\log_{10}(\text{intensity})$
dynamics determined by location in the band structure

upper band: anomalous dispersion = positive mass
\[ \Rightarrow \text{focused by nonlinearity} \]

lower band: normal dispersion = negative mass
\[ \Rightarrow \text{defocused by nonlinearity} \]

Broad Beams with Tunable Mass \( \Rightarrow \) Nonlinear Action
Nonlinear Interaction of Beams with Opposite Mass

Newton's law: Two bodies interact with equal forces of opposite direction

\[ \overrightarrow{F_1} = - \overrightarrow{F_2} \]

Interacting bodies with negative and positive masses
\[ \Rightarrow \text{acceleration into the same direction} \]

\[ |m_1| \overrightarrow{a_1} = |m_2| \overrightarrow{a_2} \]

Interacting beams with equal power, but opposite sign of dispersion
\[ \Rightarrow \text{self-accelerating bound state (diametric drive)} \]

H. Bondi, Rev. Mod. Phys. 29, 423 (1957).
A Diametric Drive in the Experiment

- acceleration obtained $\Rightarrow$ diametric drive works

- the critical velocity of the system can not be exceeded
A Diametric Drive in the Experiment

- acceleration obtained $\Rightarrow$ diametric drive works

- the critical velocity of the system can not be exceeded $\Rightarrow$ quasi relativistic system
Acceleration in a relativistic system

- velocity limited by the velocity of light

\[ x(\tau) = \frac{c^2}{a} \cosh \left( \frac{a}{c} \tau \right) \quad t(\tau) = \frac{c}{a} \sinh \left( \frac{a}{c} \tau \right) \]

\( \tau \): time in a co-moving frame
\( c \): critical velocity
\( a \): acceleration

relativistic fit
Diametric Drive in Terms of the Band Structure

\[ \text{velocity change} = \text{momentum change} = \text{frequency change} \]

acceleration terminates at the zero dispersion point
Diametric Drive in Photonic Crystal Fibers (Simulations)

- only a single zero dispersion point
- 2 pulses launched symmetrically to the zero dispersion point
- interaction via cross-phase modulation

\[
\begin{align*}
    i \frac{\partial A}{\partial z} & = \frac{\beta_A}{2} \frac{\partial^2 A}{\partial \tau^2} - \gamma_A \left( |A|^2 + 2 |B|^2 \right) A, \\
    i \frac{\partial B}{\partial z} & = \frac{\beta_B}{2} \frac{\partial^2 B}{\partial \tau^2} - \gamma_B \left( |B|^2 + 2 |A|^2 \right) B.
\end{align*}
\]

Diametric Drive in Photonic Crystal Fibers (Simulations)

Schrödinger soliton:
- propagation at anomalous dispersion (positive mass)
- attracted by the normal dispersive pulse
- accelerated towards the normal dispersive pulse

parameters:
GVD=±20 ps²/km,
T_{soliton}=1ps, E=0.8pJ
Diametric Drive in Photonic Crystal Fibers: Shape

parameters:
\( GVD = \pm 20 \text{ ps}^2/\text{km}, \)
\( T_{\text{soliton}} = 1\text{ps}, E = 0.8\text{pJ} \)

normal dispersive pulse:
- repelled by the Schrödinger soliton
- trapped by acceleration and Schrödinger soliton
Diametric Drive in Photonic Crystal Fibers: Propagation

- concept should work in fibers
- direction of acceleration: towards the normal dispersive pulse
- magnitude of acceleration: determined by pulse power
  \[ \Rightarrow \text{the stronger the pulses, the higher the acceleration} \]
Diametric Drive in Photonic Crystal Fibers: Properties

- can be excited with Gaussian pulses
- robust against collisions
- direction of acceleration / frequency shift can be chosen
- no limit of acceleration

for frequency detuning away from the zero dispersion point

⇒ perfect high power + tuneable frequency source
Diametric Drive in Photonic Crystal Fibers: Perturbations

Raman effect + 3rd order dispersion:

⇒ additional forces, but no significant changes

**temporal evolution**  
**spectral evolution**
Summary

- concepts of spatial discreteness transferred to the temporal domain
- losses can create
  - $PT$ symmetry + unidirectional invisibility
  - fractal pattern
- interaction of beams of opposite masses
  $\Rightarrow$ self-accelerating bound states

A. Regensburger et al., Phys. Rev. Lett. accepted
S. Batz and U. Peschel, Phys. Rev. Lett. accepted